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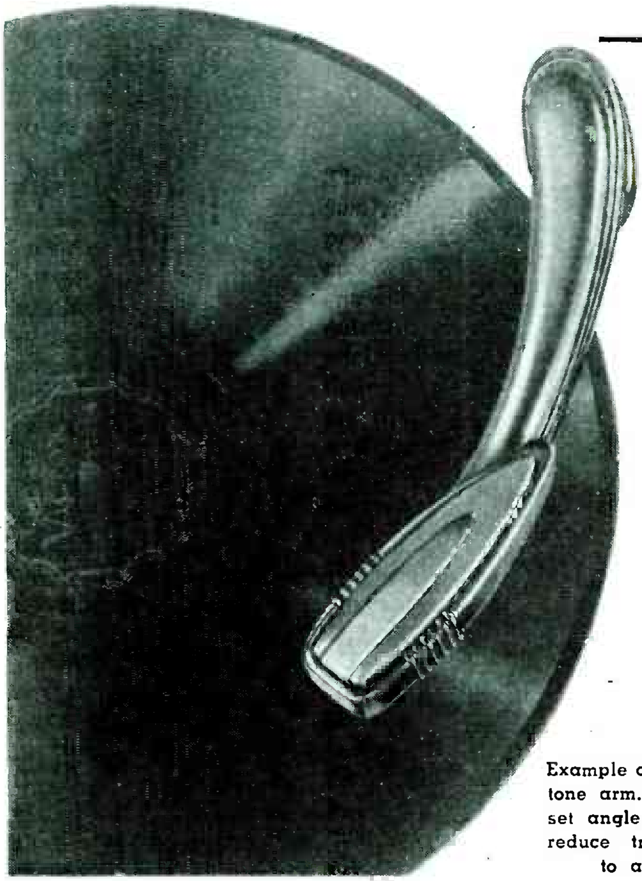
# Tracking Angle



Example of well-designed modern tone arm. Proper choice of offset angle and needle overhang reduce tracking-error distortion to a negligible value

Distortion due to tracking error can be minimized by bending the pickup arm and overhanging the needle. Equations are developed for determining optimum bending and overhang for any given conditions, and a design chart is provided for use with 12-inch records

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ALMOST EVERYONE conversant with phonograph reproduction is familiar with the fact that in a straight tone-arm the needle enters the groove laterally at an angle, which has some undesirable effects upon the record and upon the quality of reproduction. The exact nature and the quantitative amounts of these effects appear to be considerably less well known.

It is widely recognized that such effects can be corrected by offsetting the tone-arm head and adjusting its position in respect to the center of the turntable. Theory dealing with the actual amount of the necessary adjustments, however, is not generally available to tone-arm designers and users. The purpose of this article is to present to phonograph designers, in readily useful form, quantitative information regarding tracking angle and its effects, and means for minimizing these effects in modern phonograph equipment.

It is generally accepted that tracking angle produces the following effects: (a) Record wear; (b) Distortion; (c) Side-thrust upon the record grooves. Much has been written in the past ten years about these effects. Oftentimes only one of them has been emphasized at the expense of the others. This resulted in many offset arm designs which were inferior in performance to the equivalent straight arms.

Additional factors have entered the picture recently as a result of the introduction of home recording, light-weight pickups, permanent-point styli, and the widespread use of record changers. In order to deal with these subjects systematically the various effects will be treated in the following order: (1) Geometry of the tracking angle; (2) Record wear; (3) Side thrust; (4) Distortion; (5) Best arm offset; (6) Optimum arm design.

#### Definition of Tracking Angle

Let a tone arm having an effective length $l$ inches (distance from the center of the pivot to the needle point) be mounted on a motorboard with its pivot $d$ inches away from the center of the record, as shown in Fig. 1(a). When the needle point is $r$ inches away from the center of the record the following relationships exist:

$$r^2 + l^2 - 2rl \cos \theta = d^2 \qquad (1)$$

$$r^2 + l^2 - 2rl \sin \phi = d^2 \qquad (2)$$

where $\theta$ is the angle included between the lines $l$ and $r$, and $\phi$ is the tracking angle between $l$ and a line tangent to the groove at the needle point. Solving for $\sin \phi$:

$$\sin \phi = \frac{r}{2l} + \frac{l^2 - d^2}{2lr} \qquad (3)$$

It will be seen that the amount of arm overhang $D$ (swing of the needle point beyond the center of the turntable) is a significant parameter. Taking advantage of the fact that $d = l - D$, Eq. (3) may be rewritten as

$$\sin \phi = \frac{r}{2l} + \frac{2lD - D^2}{2lr} \qquad (4)$$

Equation (4) is precise in every respect, but does not lend itself to simple analytical treatment. We shall therefore make two simplifying assumptions, that $D^2$ is negligibly small compared with $2lD$, and that $\sin \phi = \phi$ in radians. When this is done, Eq. (4) can be rewritten as

$$\phi_{rad} = \frac{r}{2l} + \frac{D}{r} \qquad (5)$$

# in Phonograph Pickups

$$\phi_{deg} = 57.3\left(\frac{r}{2l} + \frac{D}{r}\right) \qquad (6)$$

The approximations tend to be mutually compensatory, and Eq. (6) provides values of $\phi$ which are correct within approximately 1 deg, as can be readily demonstrated by substituting assumed values of variables in Eq. (4) and (6). This degree of approximation is ample for the problem on hand, and it is well justified in view of the labor saving which it affords. In all the derivations which follow, $\phi_{rad}$ is conveniently used. This is multiplied by 57.3 to obtain $\phi_{deg}$ for use in final calculations.

#### Tracking Angle Curves

Figure 1(b) shows graphically the values of $\phi_{deg}$ as a function of the radius for a 7½-inch arm and for various values of arm overhang $D$. The custom in the early '30s was to use straight tone arms pivoted in such a manner that the needle point passed through the center of the record. The tracking angle corresponding to this condition is represented by curve AA for $D = 0$. It is seen that $\phi$ at the 5.75-inch radius is 22 deg, and this gradually decreases to 7.6 deg at the inner radius of 2 inches. The thought of a needle point entering the record grooves at a substantial lateral angle, together with the fact that this angle changed throughout a wide range of values and thereby involved continuous regrinding of the tip, was not especially reassuring to phonograph designers.

The most obvious answer—that of greatly lengthening the tonearm—was found to be impractical because of motorboard space limitations. A study of curves similar to those of Fig. 1(b) indicated that the remedy might lie in performing two operations: (1) Swinging the arm beyond the center of the record, as exemplified by curves EE in Fig. 1(b); (2) Bending or offsetting the head through an angle $\beta$, approximately equal to the average angle over the range of radii (this is equivalent to raising the zero-degree ordinate on the graph to any given value of $\beta$). It should be noted that $\beta$ is measured clockwise from the line connecting the needle point with the pivot point; it is not, as it is sometimes erroneously assumed, the angle between the body and the head of the tone arm.

As an example, if the needle point overhang $D$ is $\frac{13}{16}$ inch as for curve EE in Fig. 1(b), the tracking angle varies from 32 deg at 6 inches through 27 deg at 3.3 inches back to 32 deg at the 2-inch radius. Now, if the pickup head is bent at a 29.5-deg angle, the departure of the needle from tangency is reduced to only 2.5 deg on either side. The difference between the tracking angle, $\phi$, and the offset angle, $\beta$, has been termed tracking error $\alpha$, for it indicates how short of perfect the scheme is.

The above procedure for tracking angle correction (giving minimum $\alpha$) appeared to remedy all of the straight-arm objections, and it was widely adopted in many reproducers built in the years of 1938-1941. This was somewhat unfortunate, for it was shown later that minimum $\alpha$ is not a valid criterion of best tracking.

#### Record Wear

The theory behind record wear due to improper tracking angle was based upon the fact that the spherical point of a steel needle rapidly wears down to a chisel-point after the playing of the first few grooves. As the tracking angle varies throughout the playing of the record, the chisel-point turns with respect to the grooves and is constantly reground, thereby increasing record wear over and above that which would normally exist.[1,2]

With rare-metal or sapphire-tipped needles, the point remains spherical for many hundreds or even thousands of plays. There is therefore little or no justification for fearing increased record wear due to moderate amounts of tracking angle with modern playback equipment if the permanent-point needles are not employed beyond their rated life.

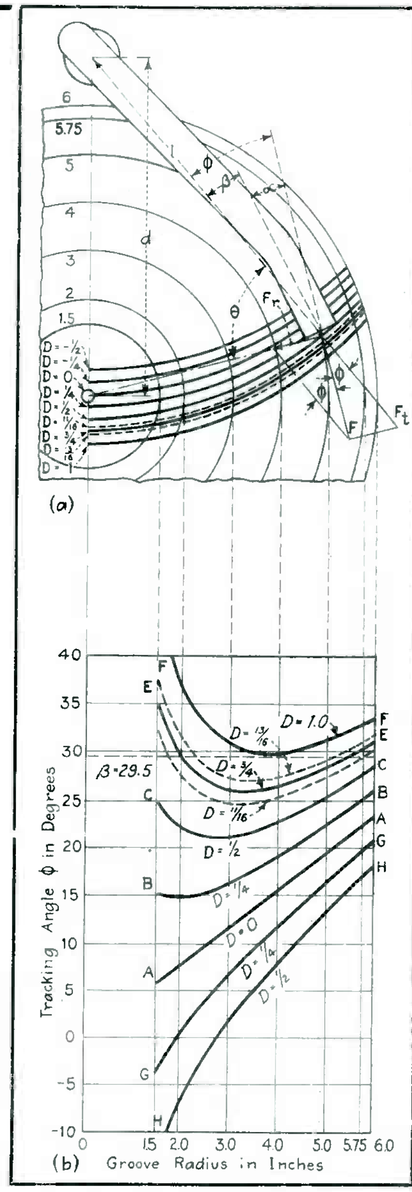


FIG. 1—Diagram showing dimensions and angles involved in tone arm design, and curves showing how tracking angle varies with groove radius for different values of needle overhang $D$. All linear dimensions are in inches

#### Side Thrust

Due to friction between the needle and the groove, there is a force $F$ upon the needle point in the direction of the line tangent to the groove. This force is quite independent of the angle which the pickup head makes with the groove, and it depends only upon the vertical needle force $F_r$ and upon the

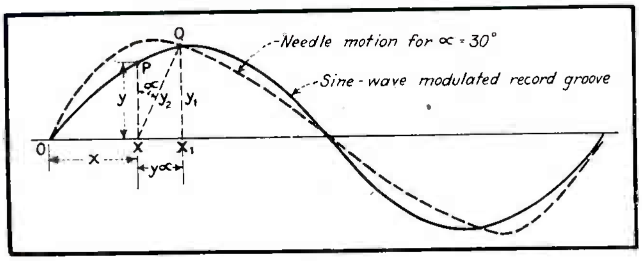


FIG. 2—Manner in which a sinusoidal wave form is distorted due to tracking error

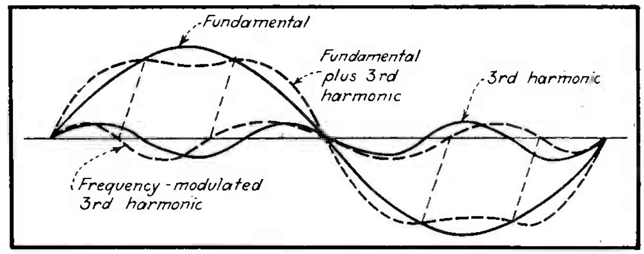


FIG. 3—Frequency modulation effect in phonograph pickup output due to tracking error

coefficient of friction $\mu$ between the needle point and the record ($F = \mu F_v$). The value of $\mu$ varies with the type of record material and also with the amount of groove modulation. For most record materials and light needle forces, $\mu$ is roughly $\frac{1}{3}$. With heavy pickups and soft records, $\mu$ may be considerably greater.

The force diagram at the needle point in Fig. 1(a) indicates the action of the frictional force. It is seen that $F$ has a tangential component $F_t$ which is borne by the arm pivot, and a radial component $F_r$ which tends to pull the pickup toward the center of the record and which is borne by the inner side of the record groove. From the force parallelogram it is seen that $F_r = F \tan \phi$. This force is not altered by any conceivable twisting of the pickup head. For any given record and pickup, $F_r$ is therefore a function only of $r$ and $D$.

Take again the case of a $7\frac{1}{2}$-inch arm with the needle passing through the center of the record, corresponding to curve AA in Fig. 1(b). The tracking angle is 22.5 deg ($\tan^{-1}$ 0.415) at the 6-inch radius and 7.6 deg ($\tan^{-1}$ 0.133) at the 2-inch radius. If $\mu$ is assumed to be $\frac{1}{3}$, the side thrust corresponding to these angles, in a 1-oz pickup, is 0.14 and 0.05 oz respectively.

If $D$ is $\frac{1}{2}$ inch as for curve CC in Fig. 1(b), the angles are 28 deg ($\tan^{-1}$ 0.53) and 22 deg. ($\tan^{-1}$ 0.40) and the corresponding side thrust is 0.18 and 0.13 oz respectively. Swinging the arm beyond the center of the record thus increases the side thrust and keeps it more or less uniform throughout the playing of the record.

It has been pointed out that moderate side thrust may not be detrimental because it helps to overcome pivot bearing friction. On the other hand, the unduly large values of $D$ employed in some instances increase $F_r$ beyond the safe point. Some phonographs cannot reproduce home-recording records because the side thrust becomes large enough to pull the tone-arm out of the groove toward the center of the record.

### Record Changer Requirements

The effect of arm placement upon the side thrust is most pronounced at inner radii of the record. This makes side thrust of special interest in connection with record changers. Some changers require a force directed away from the center of the record to actuate the tripping mechanism. In supplying this force, the tone arm is aided by the use of low and negative values of $D$. Other changers require a force directed toward the center of the record. This can be aided by the use of larger values of $D$. Such procedure may not be consistent with the conditions of minimum tracking-error distortion. In record changers, a slight compromise in distortion may be justified if reliable operation of the tripping mechanism is helped thereby.

An occasional source of greatly increased side thrust is found in some of the needles which have recently appeared on the market. Many of these needles are forwardly bent so that the effective value of $D$ is increased by $\frac{1}{8}$ to $\frac{1}{4}$ inch. In general, there are no apparent ill effects due to the use of such needles; however, instances are known when tracking has been impaired because of increased side-thrust.

The effect of sidethrust can be kept within safe limits if it is recognized that the tone arm is often called upon to track at groove radii less than 2 inches. Because of this, a minimum radius of $1\frac{3}{4}$ inches is used in all the following calculations pertaining to 12-inch records. When this precaution is observed, side thrust developed with optimum values of $D$ is quite harmless in low-weight pickups.

### Distortion

When the axis of the pickup cartridge is not tangent to the groove, the needle motion is not perpendicular to the groove, giving rise to distortion. Mathematical analysis of this distortion has been made by Baerwald[3]. The simplified derivation given below helps to bring out the essential physical factors involved and yields results which are sufficiently accurate for all intents and purposes. The solid line in Fig. 2 represents a sine-wave modulated record groove, and the dotted line is the distorted sinusoid representing the needle motion when $\alpha$ = 30 deg.

When $\alpha = 0$, the equation of needle point displacement is the same as that of the groove. (Elastic deformation of the record material and pinch effect are neglected). Assume that initially the needle point is at O and $\alpha = 0$. When the groove travels a distance $x$ (from O to X), the needle moves from O to P and its lateral displacement $y$ in inches equals

$$y = A \sin 2\pi x/\lambda \qquad (7)$$

where $A$ is maximum groove amplitude in inches, and $\lambda$ is the wavelength of groove modulation in inches. However, if the tracking error $\alpha > 0$, the needle point moves to Q instead of P; in so doing it advances horizontally a distance approximately equal to $d = y\alpha$, where $\alpha$ is in radians. This is approximate, but close enough for our purpose. Instead of being at X, the horizontal projection of the needle point is now at $X_1$. The equation of the motion described by the needle point is

$$y_1 = A \sin x_1 = A \sin \frac{2\pi}{\lambda}(x + y\alpha) \quad (8)$$

Substituting in Eq. (8) the expression for $y$ in Eq. (7) now gives

$$y_1 = A \sin\left(\frac{2\pi x}{\lambda} + \frac{2\pi A \alpha}{\lambda} \sin \frac{2\pi x}{\lambda}\right) \quad (9)$$

If the groove moves with a velocity of $V$ inches per second, then $x = Vt$ and $f = V/\lambda$ cycles per second, and

$$y_1 = A \sin\left(\omega t + \frac{\omega A \alpha}{V} \sin \omega t\right) \quad (10)$$

What is of interest is not $y_1$ but the lateral motion of the needle point, which is $y_2 = y_1/\cos\alpha$; therefore,

$$y_2 = \frac{A}{\cos\alpha} \sin\left(\omega t + \frac{\omega A \alpha}{V} \sin \omega t\right) \quad (11)$$

### Frequency-Modulation Effect

Examination of Eq. (11) shows that there is a slight increase in playback level due to the $1/\cos\alpha$ term, and frequency modulation of the signal. In Eq. (10), let $y_1 = A \sin\psi$. Differentiating $\psi$ to obtain instantaneous angular velocity $\omega_i$,

$$\omega_i = \frac{d\psi}{dt} = \omega + \frac{\omega^2 A \alpha}{V} \cos \omega t$$

$$\omega_i = \omega\left(1 + \frac{\omega A \alpha}{V} \cos \omega t\right) \quad (12)$$

The instantaneous frequency of the signal is thus modulated at a rate equal to its own frequency and with a frequency deviation of $\omega A\alpha/V$. This is of special interest when the wave is complex, consisting of a large-amplitude, low-frequency fundamental $f$ and a small-amplitude higher-frequency component $f_1$. This effect is indicated graphically in Fig. 3. It may be shown that $f_1$ is modulated at a frequency $f$ by the amount given in parentheses in Eq. (12). In a manner identical to other frequency-modulation phenomena, this gives rise to inharmonic terms having frequencies $(f + f_1)$, $(f + 2f_1)$, etc. Analysis of the nuisance value of this distortion is somewhat complex[3] but the overall results may be roughly estimated from harmonic analysis of Eq. (11). This equation may be expanded algebraically into its harmonic components:

$$y_2 = \frac{A}{\cos\alpha}\left[\sin \omega t \cos\left(\frac{\omega A \alpha}{V} \sin \omega t\right) + \cos \omega t \sin\left(\frac{\omega A \alpha}{V} \sin \omega t\right)\right]$$

It will be shown later that normally $\omega A\alpha/V < 0.06$ radian; therefore, with an error not in excess of 1 percent, one may state that

$$\sin\left(\frac{\omega A \alpha}{V} \sin \omega t\right) \simeq \frac{\omega A \alpha}{V} \sin \omega t$$

$$\cos\left(\frac{\omega A \alpha}{V} \sin \omega t\right) \simeq 1$$

Substituting these simplifications in the expansion of Eq. (11) gives

$$y_2 = \frac{A}{\cos\alpha}\left(\sin \omega t + \frac{\omega A \alpha}{V} \sin \omega t \cos \omega t\right)$$

$$y_2 = \frac{A}{\cos\alpha}\left(\sin \omega t + \frac{\omega A \alpha}{2V} \sin 2\omega t\right) \quad (13)$$

### Second Harmonic Distortion

Equation (13) consists of a fundamental and a double-frequency term, representing second harmonic distortion. Distortion is given by the modulus of the second term in parentheses:

$$\text{Percent 2nd harmonic (amplitude)} = \frac{\omega A \alpha}{2V} \times 100 \quad (14)$$

This is on an amplitude basis; on a velocity basis harmonics are accentuated in proportion to the frequency, hence

$$\text{Percent 2nd harmonic (velocity)} = \frac{\omega A \alpha}{V} \times 100 \quad (15)$$

For ease in interpreting Eq. (15) let the angular velocity $\omega_r$ of the record be $2\pi$ times speed in rps. Then the linear groove velocity is proportional to the radius, and hence $V = \omega_r r$. Inserting this in Eq. (15) gives

$$\text{Percent 2nd harmonic} = \frac{\omega A \alpha}{\omega_r r} \times 100 \quad (16)$$

From Eq. (16) the following is concluded:

(a) Distortion is proportional to the ratio between tracking error $\alpha$ and groove radius $r$. The ratio $\alpha/r$ may therefore be considered as an index of distortion. If distortion is not to exceed a given value throughout the playing of the record, this value of $\alpha/r$ must not be exceeded.

(b) Differentiating Eq. (7) with respect to time, it is found that maximum groove modulation velocity equals $\omega A$. Distortion is therefore directly proportional to the velocity of groove modulation.

(c) Distortion is inversely proportional to the speed of the record. For equal distortion, more careful tracking angle correction is required in 33-rpm discs than in 78-rpm discs, other factors being the same.

### Example of Distortion

As a specific example of distortion due to straight arms, assume a modulation amplitude of 0.0017 inch and a frequency of 250 cps, equivalent to a modulation velocity $\omega A$ of 2.67 inches per second. These conditions are chosen because they constitute a maximum velocity on an Audiotone test record which has a constant amplitude characteristic from 50 to 250 cps and a constant velocity characteristic thereafter. In a 78-rpm record $\omega_r = 2\pi(78/60) = 8.16$ radians per second. Now, take the case of a

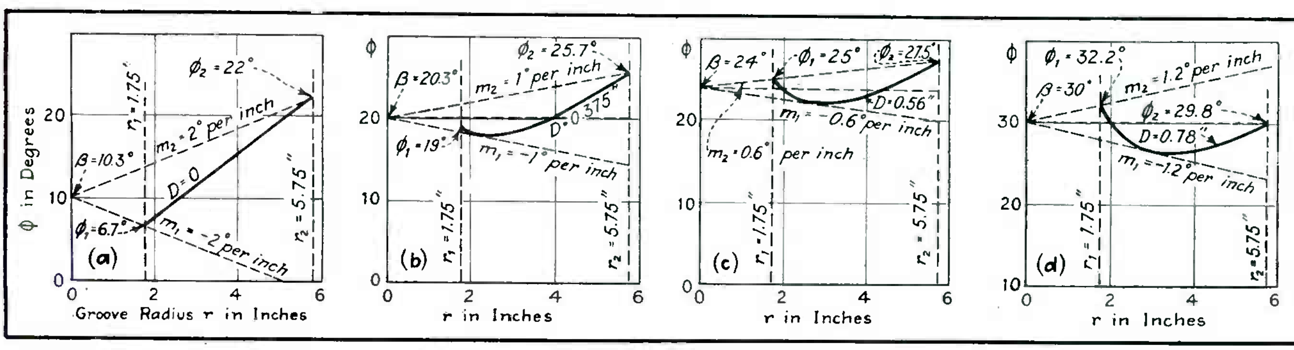


FIG. 4—Graphical procedure for determining the best arm offset angle for various overhang values ranging from no overhang (a) to 0.78-inch overhang (d)

straight pickup with the needle passing through the center of the record. From Eq. (5), tracking angle (and, in this case, tracking error $\alpha$) equals $r/2l$ radians and $\alpha/r$ equals $1/2l$ radians per inch. Substituting these values into Eq. (16), it is found that, in the constant-velocity portion, distortion is constant at all radii and equals 2.2 percent. This constancy is to be expected from considerations made in connection with Eq. (16).

It is doubtful that this amount of distortion is significant in the majority of phonograph apparatus. However, questions arise regarding distortion on records with levels higher than the Audiotone.

The relative levels of modulation velocity may be determined by examination of the width of reflected light patterns.[4] Comparison of pattern width of commercial pressings with the Audiotone record indicates that the former exhibit modulation levels 2 to 2½ times greater than the latter. Therefore, in playing commercial recordings, a tracking error distortion of 4 to 6 percent may be expected with straight-arm reproducers. These values are comparable to other distortions found in phonograph reproduction; no one should expect, therefore, startling improvements in fidelity due to elimination of tracking error. However, even a moderate amount of distortion added on top of other distortions in the system may become greatly annoying, especially in extended-range systems.

### Determining Arm Offset

To determine the proper arm offset for any given $D$, we use tracking angle curves like those in Fig. 1(b). Curves for four selected values of $D$ have been drawn in Fig. 4. In each instance we shall find the value of $\beta$ for the least $\alpha/r$, corresponding to the least distortion possible with the given $D$. In so doing we shall also have solved the converse problem: "What should $D$ be for an arm of given $\beta$"?

In Fig. 4(a), $\phi$ is shown for the condition of $l$ = 7.5 inches and $D$ = 0. Tracking angle $\phi$ varies from 22 deg at the 5.75-inch radius to 6.7 deg at the 1.75-inch radius. For straight arms $\alpha = \phi$, and $\alpha/r$ = 22/5.75 = 3.8 deg per inch. This has been found to produce distortion of from four to six percent.

Consider now the arm which is offset at an angle of 10.3 deg. Here $\alpha$ at 5.75-inch radius is 22 − 10.3 = 11.7 deg; $\alpha$ at 1.75-inch radius is 10.3 − 6.7 = 3.6 deg. For both radii, $\alpha/r$ is now 2 deg per inch. Offsetting the arm 10.3 deg has reduced distortion by a ratio of almost 2:1.

An offset of 10.3 deg produces lowest distortion in a 7.5-inch arm, if $D$ equals zero. This can be easily verified by repeating the above calculations for values of $\beta$ other than 10.3 deg. If one extends two lines from the 10.3-deg point on the zero-inch ordinate to the two terminals of the $\phi$ curve, as shown in dotted lines in Fig. 4(a), it becomes apparent that the slopes $m_1$ and $m_2$, in degrees per inch, are equal and opposite. It is not difficult to reason out that this is a necessary and sufficient condition for least $\alpha/r$. Therefore, in determining the arm offset for any given $D$, the following procedure may always be employed:

Draw the $\phi$-curve corresponding to the given $l$ and $D$ between the limits $r_1$ and $r_2$. Extend straight lines just bounding the $\phi$-curve on each side from a point $\beta$ on the zero-inch ordinate such that the slopes $m_1$ and $m_2$ are equal and opposite. Angle $\beta$ is then the offset angle yielding least tracking error distortion. This procedure is valid for any and all values of $l$, $D$, $r_1$ and $r_2$.

In the instance when the slope lines touch the two extremities of the $\phi$-curve (as in the above example), $\beta$ can also be found analytically without any difficulty. Tracking angles $\phi_1$ and $\phi_2$ in radians at radii $r_1$ and $r_2$ are

$$\phi_1 = r_1/2l + D/r_1 \qquad (17)$$

$$\phi_2 = r_2/2l + D/r_2 \qquad (18)$$

If the two slope lines are equal,

$$(\beta - \phi_1)/r_1 = (\phi_2 - \beta)/r_2 \qquad (19)$$

Substituting Eq. (17) and (18) into (19) and solving for $\beta$, radians,

$$\beta = \frac{\left(\frac{1}{r_1^2} + \frac{1}{r_2^2}\right) D + \frac{1}{l}}{\frac{1}{r_1} + \frac{1}{r_2}} \qquad (20)$$

If $D$ = 0,

$$\beta = \frac{1}{l\left(\frac{1}{r_1} + \frac{1}{r_2}\right)} \qquad (21)$$

This gives the best arm offset when the needle passes through the center of the record. If $\beta = 0$,

$$D = -\frac{1}{l\left(\frac{1}{r_1^2} + \frac{1}{r_2^2}\right)} \qquad (22)$$

This indicates that straight arms should be underhung for least distortion.

Equation (20) represents a family of straight lines. This family, for 12-inch discs, is given in Fig. 5, and it extends from the 0-deg ordinate to the inclined dotted line labelled "Limit of simple placement equation."

For values of $D$ greater than indicated by the limit line, Eq. (20) is no longer valid because the lower slope-line $m_1$ touches the $\phi$-curve at a point other than $\phi_1$. Such a situation is shown in Fig. 4(b) for a 7.5-inch arm when $D$ = 3/8 inch. For the least $\alpha/r$, $\beta$ is now found to be 20.3 deg by employing the graphical method given before. The analytical relation between $\beta$ and $D$ is given in this instance by

$$D = \frac{r_2}{2}\left(\frac{r_2}{l} - \beta\right)\left(\sqrt{1 + \frac{\beta^2}{\left(\frac{r_2}{l} - \beta\right)^2}} - 1\right) \qquad (23)$$

The derivation of this equation is not difficult, but it is tedious enough to be relegated to the Appendix. The family of curves represented is a set of curved lines which are extensions of the straight lines of Eq. (20).

A careful study of Fig. 1(b) or Fig. 4 reveals that progressive increase of $D$ (and use of the corresponding best $\beta$) diminishes the distortion index $\alpha/r$. This is shown by the diminishing angle between the two bounding lines $m_1$ and $m_2$. In the instance of $D$ = 3/8 inch given above, $\alpha/r$ is 1.0 deg per inch, which is a 4:1 decrease compared with a straight arm.

As $D$ is increased further, the point is finally reached when $\alpha/r$ is minimum and, distortionwise, optimum arm design is achieved. This is shown in Fig. 4(c) where $\alpha/r$ is only 0.6 deg per inch. In Fig. 5, the relation between $\beta_{opt}$ and $D_{opt}$ is given by the straight dotted line labeled "Line of optimum arm design."

### Example of Excessive Offset

Beyond the optimum point, the upper slope-line $m_2$ passes through

$\phi_1$, and the angle between $m_1$ and $m_2$ again increases—and so does distortion and side thrust. This condition occurs in a number of pre-war arms.

Figure 4(d) shows proper placement of a 7½-inch arm having an offset angle of 30 deg. The best value of $D$ here is 0.78 inch.

The derivation of the best placement equation is given in the Appendix. For least distortion, the relation between $\beta$ (radians) and the corresponding $D$ (inches) is as follows:

$$D = \frac{R_1}{2}\left(\beta - \frac{r_1}{l}\right)\left(\sqrt{1 + \frac{\beta^2}{\left(\beta - \frac{r_1}{l}\right)^2}} + 1\right) \quad (24)$$

This equation represents a series of lines which are practically straight and which are shown in Fig. 5 extending beyond the "Line of optimum arm design." Figure 5 can be used, therefore, as a universal chart for properly locating arms with pre-determined $\beta$, or for determination of $\beta$ in arms requiring a given amount of overhang because of tracking reasons.

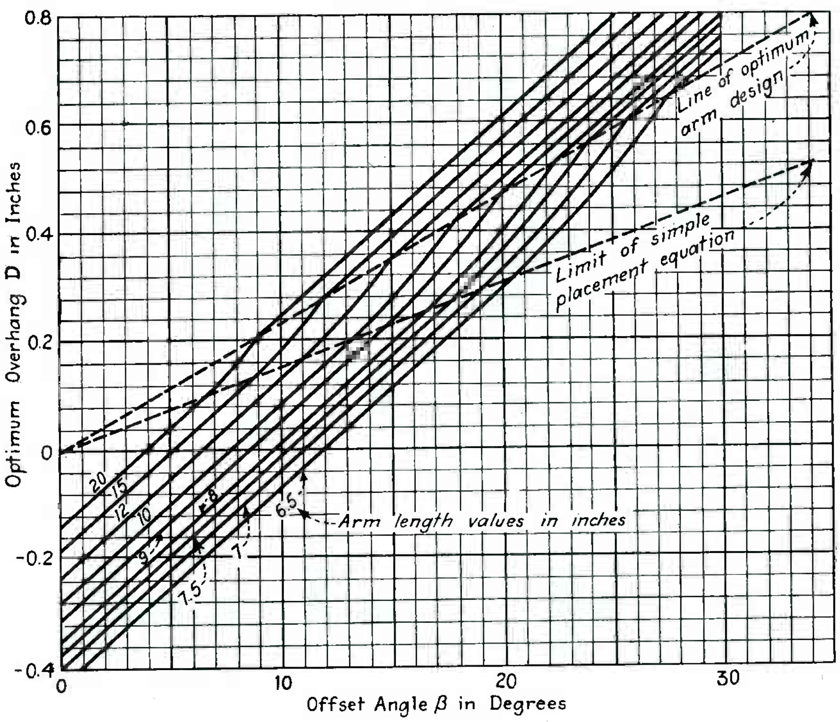


**FIG. 5—Design chart for tone arms used with 12-inch records, in which minimum groove radius $r_1$ is 1.75 inches and maximum groove radius $r_2$ is 5.75 inches**

#### Optimum Arm Design

Keeping in mind the procedure for obtaining best $\beta$, the following conditions are fulfilled when $\alpha/r$ is minimum: (1) Both extremities of the $\phi$-curve touch the upper slope-line $m_2$; (2) The lower slope line $m_1$ is equal and oppositely slanted ($m_1 = -m_2$) and just touches the lower side of the $\phi$-curve.

This situation is shown in Fig. 4(c). The relation between $l$, $r$, $r_2$, $D$, and $\beta$ corresponding to this instance will now be derived.

The slope of the line tangent to the $\phi$-curve and passing through $\beta$ is given in the Appendix as

$$m = 1/2l - \beta^2/4D \text{ radians per inch} \quad (25)$$

From condition (1) above,

$$\frac{\phi_1 - \beta}{r_1} = \frac{\phi_2 - \beta}{r_2} \quad (26)$$

Substituting values of $\phi_1$ and $\phi_2$ from Eq. (17) and (18) and solving for $\beta$ gives

$$\beta = D\frac{r_1 + r_2}{r_1 r_2} \quad (27)$$

From condition (2), $\beta - mr_1 = \phi_1$. Substituting the values of $\beta$ from Eq. (27), $m$ from Eq. (25), and $\phi_1$ from Eq. (17) and solving for $D$,

$$D = \frac{r_1^2}{l\left[\frac{1}{4}\left(1 + \frac{r_1}{r_2}\right)^2 + \frac{r_1}{r_2}\right]} \text{ inches} \quad (28)$$

Substituting this expression for $D$ in Eq. (27) now gives

$$\beta = \frac{r_1\left(1 + \frac{r_1}{r_2}\right)}{l\left[\frac{1}{4}\left(1 + \frac{r_1}{r_2}\right)^2 + \frac{r_1}{r_2}\right]} \text{ radians} \quad (29)$$

The values of $\beta$ and $D$ as given by Eq. (28) and (29) provide the minimum distortion attainable in a pivoted tone arm, and they should always be employed in tone-arm design unless this is not feasible because of other considerations. The reduction in distortion over the straight-arm situation is roughly 6:1. This renders tracking error distortion completely negligible.

#### APPENDIX

To derive the equation of arm placement in the region of tangency, it is first necessary to find the slope $m_1$ of the lower slope-line tangent to the $\phi$ curve and passing through the point $\beta$ on the zero-inch ordinate. The slope of the line connecting $\beta$ with any point on the $\phi$-curve is

$$m = \frac{\phi - \beta}{r} = \frac{\phi}{r} - \frac{\beta}{r} \quad (30)$$

Substituting the value of $\phi$ from Eq. (5) gives

$$m = \frac{1}{2l} + \frac{D}{r^2} - \frac{\beta}{r} \quad (31)$$

The point of tangency occurs when the slope is minimum; differentiating Eq. (31) gives

$$\frac{dm}{dr} = \frac{\beta}{r^2} - \frac{2D}{r^3} \quad (32)$$

For minimum slope $m_1$, $dm/dr = 0$, and

$$\beta = 2D/r \quad (33)$$

Substituting Eq. (33) in Eq. (31),

$$m_1 = 1/2l - \beta^2/4D \quad (34)$$

In order to fulfill the procedure for least $\alpha/r$, $m_2$ must equal $-m_1$, or

$$m_2 = \beta^2/4D - 1/2l \quad (35)$$

But the upper slope $m_2$ equals

$$m_2 = (\phi_2 - \beta)/r_2 \quad (36)$$

Substituting $\phi_2$ from Eq. (18) and $m_2$ from Eq. (35), Eq. (23) is obtained.

Beyond the point of optimum arm design, $m_2$ makes contact with $\phi_1$ instead of $\phi_2$. Equation (36) becomes

$$m_2 = (\phi_1 - \beta)/r_1 \quad (37)$$

Substituting $\phi_1$ from (Eq. (17) and $m_2$ from Eq. (35), Eq. (24) follows.


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